Simple Climate Model

Rodger Farley Feb 2, 2013

Presented here is a simple single point bulk temperature radiant heat transfer model of the Earth. No accounting is made for the geo-nuclear heat input (radiogenic) from the Earth's core, or transient effects due to heat capacity storage. The basic heat balance equation is derived with astronomical variations from Earth's orbit included.

Solar Constant

The solar irradiance constant is the radiant energy flux at 1 AU (astronomical unit = 1.0 mean Earth orbit radius). With the full solar spectrum it has a flux value of 1361 watts per square meter. There is a very long term slow increase over the millions of years (200 million years ago it was 3% less, and 4 billion years ago 25-30% less!), but we will only mention the short period fluctuations. They are the solar max/sunspot cycle of 11 years (Schwabe), the 88 years Gleisberg cycle, the 208 years DeVries cycle, and the 1,000 years Eddy cycle. The max yearly avegare variation is on the order of 0.1% to 0.2%, but over the centuries it has varied from 0.2% to 0.6%.

Solar irradiance at the Top Of the Atmosphere (TOA)

Earth has a slightly elliptical orbit (orbital eccentricity = 0.0167019), and since the solar energy propagates outward on a spherical wave front, the energy flux changes by a $1/R^2$ term where R is the distance between the Earth and the Sun. This accounts for a much greater change in the solar flux arriving at Earth over the course of a year as opposed to solar constant variations. A simple Keplarian orbital model can determine this variation, but to make things even simpler the variation can be approximated with a cosine function about an average value according to the day number (1 thru 365, Jan 1 = day 1):

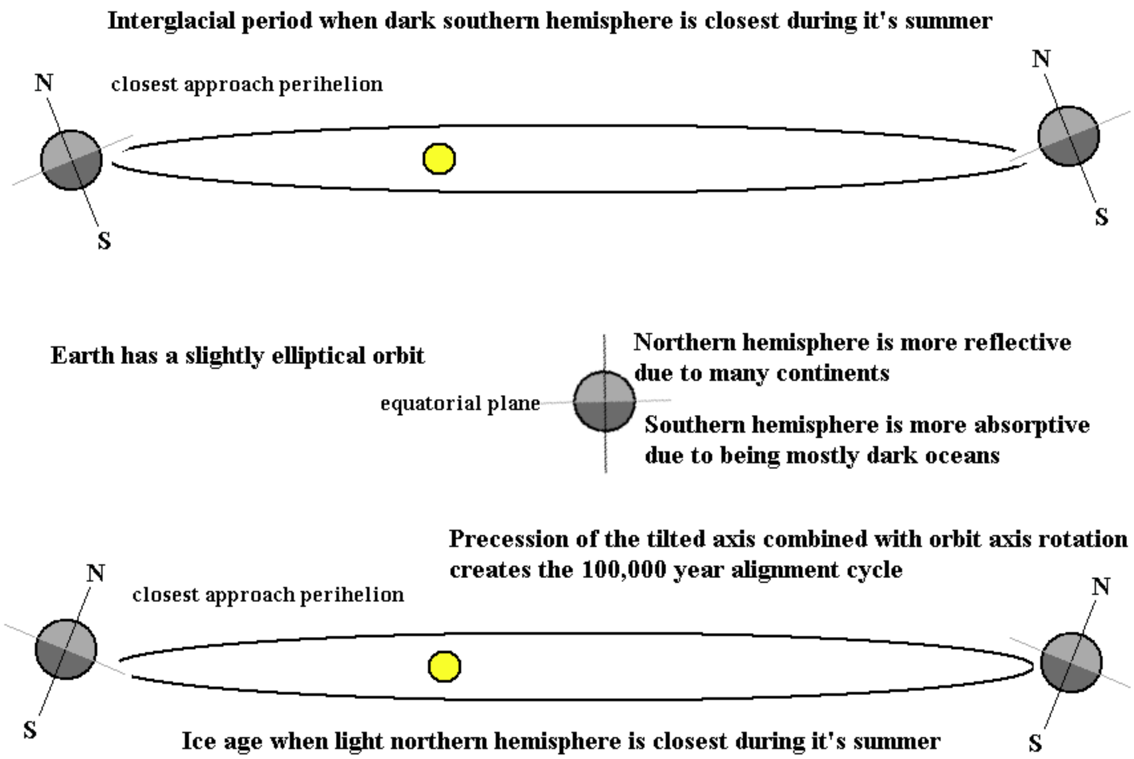
$T_{epoch} := 3020$ years past perihelion alignment with southern hemisphere summer

$d_{off} := T_{epoch} \cdot \frac{365}{100000}$ $d_{off} = 11$ number of days difference between perihelion and solstice based on 100,000 year cycle between ice ages

$SolarConstant := 1361$ watts / m^2 at 1 AU

$I_o := 1.0005 \cdot SolarConstant + 45 \cdot \cos\left(2 \cdot \pi \cdot \frac{day + 9 - d_{off}}{365}\right)$ Solar insolation at TOA at the particular day number, 1 = Jan 1, W/m^2

This formulation includes the phasing between the seasonal position of the tilted Earth axis and the point of orbital perihelion (closest approach). Note that perihelion today occurs on January 2nd, about 11 days past the solstice. Around 3000 years ago the perihelion coincided with the southern hemisphere's summer solstice. Ever since then the Earth has been drifting away past this point of max solar input coinciding with best alignment with the most absorptive hemisphere (the southern hemisphere is a dark/absorptive ocean hemisphere while the northern hemisphere is filled with continents and is more reflective). After a number of millennia from now the Earth will slip back to an ice age repeating the 100,000 year cycle. Right now the northern hemisphere winters are moderated by the greater than usual southern ocean warming due to the solstice/perihelion alignment and relatively high absorptivity of oceans. During an ice age the southern ocean loses its favorable alignment and cannot add as much heat to the oceans to moderate the northern hemisphere's winter. The greater snow fall that sticks to ground (northern hemisphere) stays and changes the Earth's albedo, reflecting more sunlight away, adding to the direction of getting cold for ~85,000 years.



Radiative heat transfer equations applied to the Earth

A short refresher on heat transfer via thermal radiation is necessary at this point. Solid opaque objects have optical properties of absorption, reflection, and emissivity. How well a surface can absorb solar energy is denoted as absorptivity α, how well it reflects solar energy is reflectivity r, and the efficiency of how a surface emits infrared energy is emissivity ε. The parameters α, r , and ε are expressed as fractions such that:

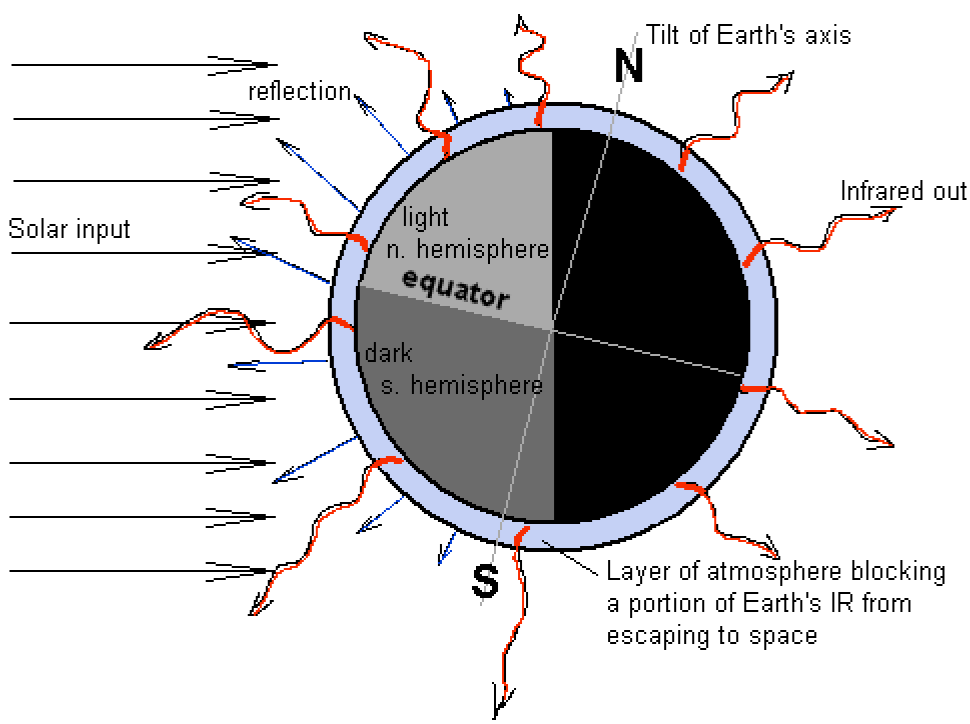
$$1 = r + \alpha \quad \text{for an opaque object, and } 0 < \varepsilon < 1$$

According to the Stefan-Boltzmann relationship, any surface with a temperature T above absolute zero emits radiant energy flux q to a fourth power of temperature:

$$q = \varepsilon \cdot \sigma \cdot T^4 \qquad \text{IR flux, Watts/m}^2$$

Where σ = Stefan-Boltzmann constant (5.67×10^{-8} W/ (m^2K^4)), ε is the surface emissivity, and T is in degrees Kelvin. A perfect emissivity of $\varepsilon = 1$ is known as a black body. With less than perfect emission qualities the IR flux will be less than a black body. For more or less room temperature objects this is in the infrared frequency band. The majority of the Sun's radiant energy is in the visible range plus near infrared, with a dash of ultraviolet. This is intercepted by the Earth's projected area (πR_{earth}^2) and the intensity varies by the distance away from the Sun as discussed earlier. Earth rejects radiant energy in two ways; it can reflect a portion of the incoming solar radiant energy, and it emits its own infrared energy due to the temperature of its full spherical surface ($4\pi R_{earth}^2$). To complicate matters, the atmosphere can reabsorb some of that radiating infrared due to the presence of water, CO_2, and methane in the air. With this effect, the atmosphere has a transmission factor of about 63%, where 37% of the surface IR is reabsorbed. We call this atmospheric transmission efficiency τ_{atm} and nominally = 0.63. High-altitude balloon flights have made similar measurements. The growth of greenhouse gasses can lower the transmission factor.

For thermal equilibrium, the total absorbed energy captured by the Earth is equal to the total rejected energy by the Earth. In other words, the Earth will attain a final temperature such that the infrared (IR) energy blaring out to space from its hot surface exactly cancels out the solar energy it is absorbing. At that point there is a balance of energies (energy in = energy out) which occurs at a specific surface temperature. The reality is that there are complex interactions between land, air, and sea where energy can be transferred, or stored and later released. The equilibrium argument is close enough and represents the worst the temperatures could get. In any case, over the course of a year, the energy stored vs. energy released averages out to zero from these sources.



Albedo is the reflective quality of the Earth's combined surface and atmosphere as seen by a far away observer. It is usually reported as ~ 0.3 (30% of solar radiant energy reflected away). So with the absorption equal to 1- reflectance, it is on average $\alpha \sim 0.7$. The average surface IR emissivity is ~ 0.96 (96% efficient at emitting infrared energy).

$albedo := 0.3$	Earths effective average reflection coefficient
$\alpha := 1 - albedo$	Earths solar absorption coefficient
$\varepsilon := 0.96$	Earths IR average emissivity at the surface
$\tau_{atm} := 0.63$	The atmosphere's transmission factor for surface generated IR flux

The solar absorbed radiant energy Q_{in} is equal to the flux at the top of the atmosphere Io times the absorptivity times the intercepted circular area:

$$Q_{in} = I_o \cdot \alpha \cdot \left(\pi \cdot R_{earth}^{2}\right)$$

Solar heat intercepted and absorbed by earth's projected area, watts

The reflected energy is accounted for in the absorption coefficient (α = 1- reflectance).

The infrared energy rejected to space from Earth Q_{out} is equal to the black-body flux (σT^4) times the emissive efficiency ε times the transmission factor thru the atmosphere times the entire surface area of the spherical Earth.

$$Q_{out} = \sigma \cdot T_{earth}^{4} \cdot \varepsilon \cdot \tau_{atm} \cdot \left(4 \cdot \pi \cdot R_{earth}^{2}\right)$$

Infrared heat emmited by the earth total surface area at the top of the atmosphere, watts

The surface emitted IR flux is $\sigma \cdot T_{earth}^{4} \cdot \varepsilon$

What finally passes thru the atmosphere to space has to be multiplied by the IR transmission factor τ_{atm}

Setting the two energies equal to each other for thermal equilibrium we get:

$$I_o \cdot \alpha \cdot \left(\pi \cdot R_{earth}^{2}\right) = \sigma \cdot T_{earth}^{4} \cdot \varepsilon \cdot \tau_{atm} \cdot \left(4 \cdot \pi \cdot R_{earth}^{2}\right)$$

$Q_{in} = Q_{out}$ (1st law of thermodynamics)

Solving for Earth temperature T_{earth} we get the final equation:

$$T_{earth} = \left(\frac{I_o}{4 \cdot \sigma \cdot \tau_{atm}} \cdot \frac{\alpha}{\varepsilon}\right)^{\frac{1}{4}}$$

degrees Kelvin

Note that the radius of the Earth is no longer in the equation, as the ratio formed is that the spherical surface area is 4 times greater than the projected circular area, regardless of the actual size.

When we plug in the values for α, ε, and τ_{atm}, and average the solar flux Io over a year (1357.33 watts/ m^2), we get an average global temperature of **288 Kelvin = 15C**. This is what is reported in the literature.

Sensitivity of temperature to changes in the variables

The Earth's systems are very complex with positive and negative feedback. By that I mean higher temperatures can melt polar caps which exposes dark oceans to absorb even more energy (positive feedback), but at the same time warmer oceans means more evaporation and cloud formation resulting in more reflection thus cooling (negative feedback). But then again more clouds means less IR transmission escaping to space as water vapor blocks upwelling IR, thus making it warmer. There is also an effect with galactic cosmic rays. Cosmic rays ionize in the lower atmosphere creating condensation nuclei for cloud formation. During sun spot activity when the sun has 0.1% greater output, its magnetism increases as well, which diverts the galactic cosmic rays from Earth. That means fewer clouds, so less reflection, so a little warmer than what just the 0.1% variation alone would accomplish. As I said, it's complicated (add to that aerosols and particulates suspended in the atmosphere).

However, we can determine the single variable sensitivity of the system by keeping all variables constant except for one which we play around with. Assuming the surface emissive efficiency stays the same at 0.96, we ask what changes in the solar reflection, or atmospheric IR transmission, or solar constant would be necessary to produce a 2 degree increase in average temperature?

Using the equilibrium temperature equation and only varying one parameter at a time we get these results:

Albedo (reflectivity) would have to decrease from 0.3 to 0.28 (6.7% decrease). This is not likely as a warming planet would create more clouds increasing the reflectivity, not decreasing it. Aerosols and suspended particulates can not only block some IR but are also more absorptive to solar input but can also be more reflective. These effects are not treated here (that's where the more complex models come in)

IR atmospheric transmissivity would only have to decrease from 0.63 to 0.621 (1.43% decrease). This makes it a rather sensitive parameter where effects from CO_2, methane, clouds and suspended particulates/aerosols have a hand in effecting changes.

The solar constant would have to increase from 1361 to 1400 watts per square meter (~2.86% increase). This has a medium influence, but is way beyond any measured changes in the solar constant, which are more in the 0.1% to 0.2% range; an order of magnitude difference to what would be required for a 2 degree increase in temperature.

Conclusions

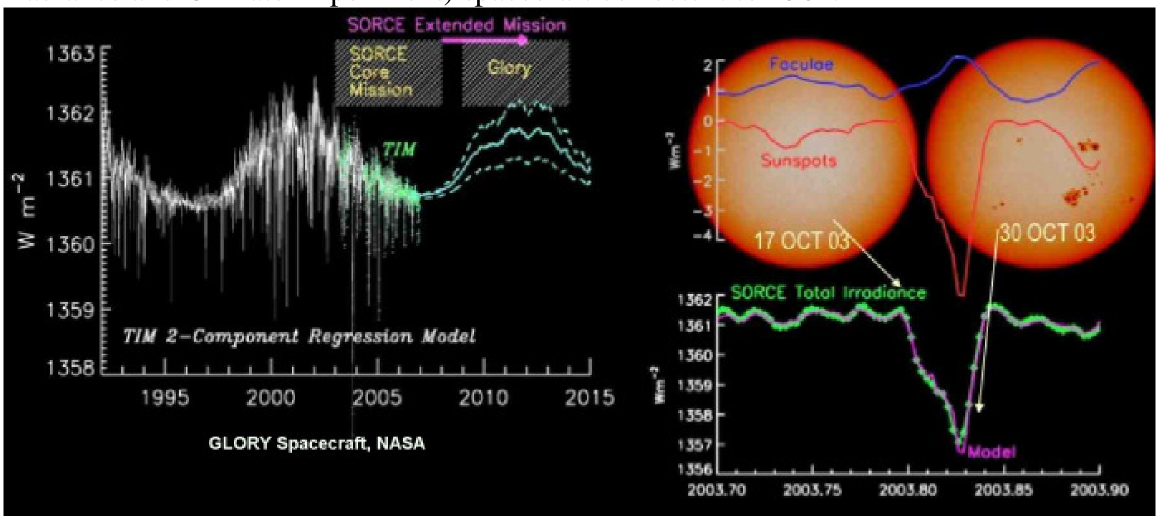
The most sensitive parameter to changes in the average Earth surface temperature is the atmospheric infrared transmission factor τ_{atm}. This is affected mostly by the water, CO_2, and methane content in the atmosphere. The CO_2 levels have been going up steadily since the industrial revolution, and are at unprecedented levels never before measured in ice core samples that go back to over 400,000 years (4 ice-age cycles). Man-made CO_2 from fossil fuel burning combined with the destruction of CO_2 “sponges” like the rain forests and saturated oceans has synergistically conspired together to explain the enormous spike of CO_2 levels. Add to that the melting of the Arctic permafrost which exposes thawing peat-moss, in which the decay by bacteria is producing enormous quantities of methane. Molecule for molecule, methane is about 20x stronger than CO_2 in trapping infrared. In geological time, all this is happening in an eye blink, further evidence to suggest man-made activity. As the radiant energy balance equation shows, only a small change is necessary to raise temperatures significantly, and this is the most likely largest contributor of global warming.

However, the measured and inferred changes in the solar constant have had historical associations with climate change. Its effects are not insignificant probably due to the feedback mechanisms that can magnify the changes (such as galactic cosmic rays). But on a one-for-one comparison to the IR transmissivity, the IR emission has a greater influence on temperature. Current global warming no doubt has a component due to the rising solar constant trend (now leveled off since mid 1970’s) and can explain rising temperatures in the early-mid 20th century, but cannot explain the rapid rise in the last 40 years. Man has lowered the atmospheric IR transmission with his industrial activities to support large populations combined with environmental destruction.

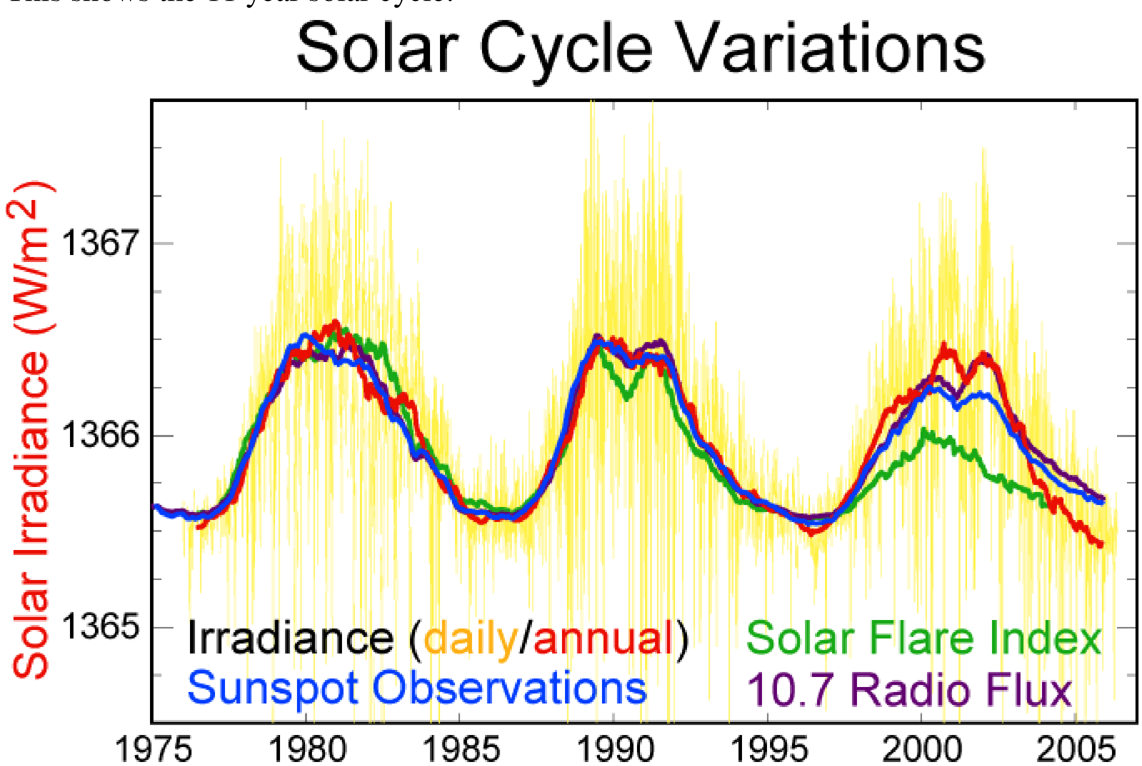
It is interesting to contemplate that on the time scale of hundreds of years that global warming and environmental destruction are the current threats to the biosphere. But in the time frame of thousands of years it is ice age that is the major threat to the planet. Earth has been in a season of glaciations for the last 2.5 million years ever since the Indian sub-continent crashed into Asia and formed the Himalayas. To explain that statement, most of our CO_2 has over the millions of years been slowly and constantly being locked away in limestone, a result of chemistry between plants, animals, rocks, and CO_2. The enormous amount of new exposed rock to be weathered and washed into the sea contributed to geologically-rapid absorption of the atmospheric CO_2. The Earth has had for the most part over the millions of years a balance between the always increasing solar output and the always decreasing level of greenhouse action. This has moderated temperatures, even though hundreds of millions of years ago the atmosphere may have had up to 3-10 x more CO_2 concentration than present day. The Sun had 3% less output at the same time too, so the balancing act has been going on for some time, but it won’t last forever. At some point complex life will be unsustainable as the open-loop thermal balancing act gets out of sync with the natural progressive forces of limestone formation.

Appendix 1

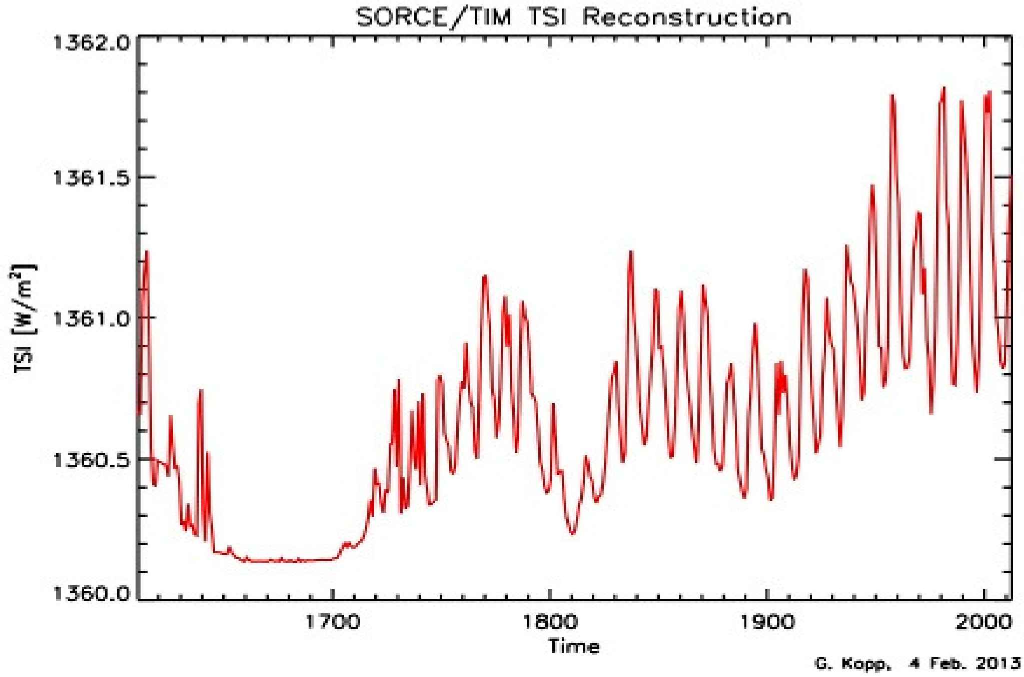
Solar Constant
Getting the exact number has been difficult over the years with older instruments indicating 1366 watts per square meter, but the latest from NASA's SOURCE (SOlar Radiance and Climate Experiment) spacecraft corrects it to 1361.



This shows the 11 year solar cycle:



Historical reconstruction of Total Solar Irradiance from Wang, Lean, and Sheely:



This next graph shows the solar constant having an increasing trend for the last several centuries. It shows a 0.18% increase since the beginning of the industrial revolution.

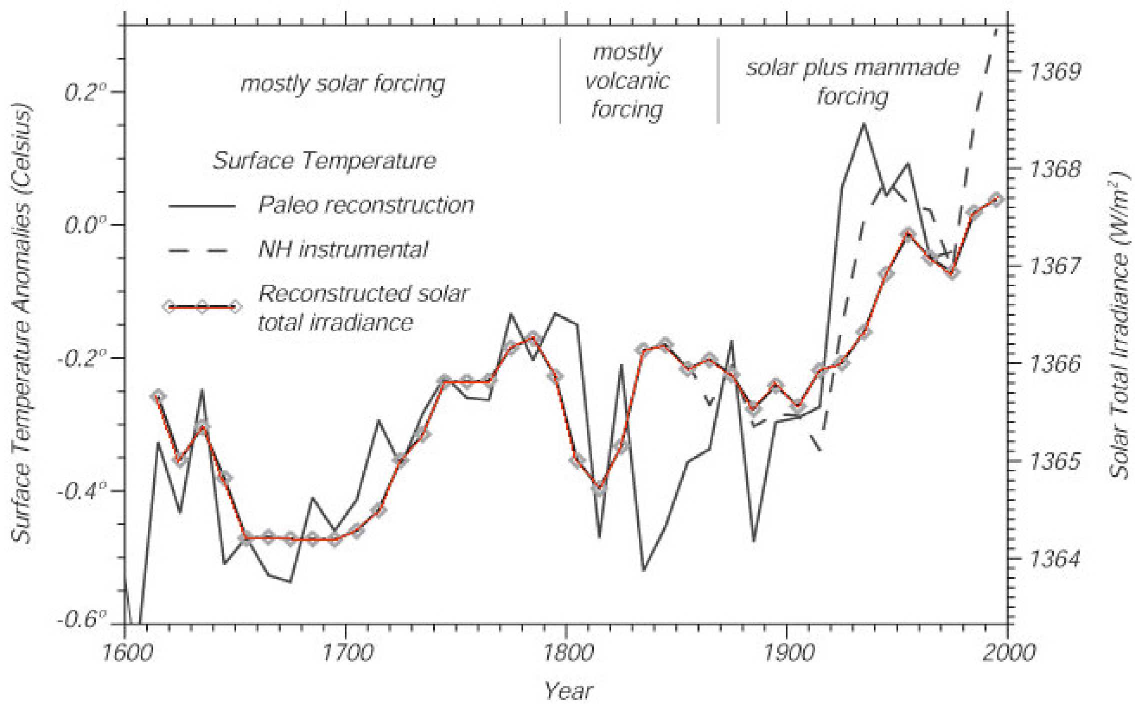


Figure 5.14 Changes in solar constant (total solar irradiance) and global mean temperature of Earth's surface over the past 400 years. Except for a period of enhanced volcanic activity in the early 19th century, surface temperature is well correlated with solar variability. From Lean, personal communication.

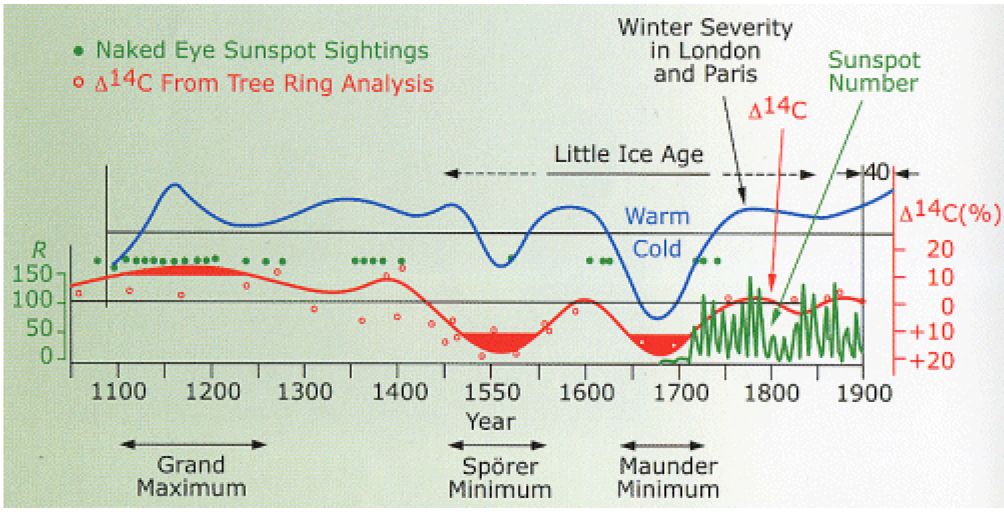
From the Department of Oceanography, Texas A&M University
Robert H. Stewart

Correlations from historical climate and solar activity

From Science.Nasa.Gov, 2003:

The intensity of the Sun varies along with the 11-year sunspot cycle. When sunspots are numerous the solar constant is high (about 1367 W/m^2); when sunspots are scarce the value is low (about 1365 W/m^2). Eleven years isn't the only "beat," however. The solar constant can fluctuate by ~0.1% over days and weeks as sunspots grow and dissipate. The solar constant also drifts by 0.2% to 0.6% over many centuries, according to scientists who study tree rings.

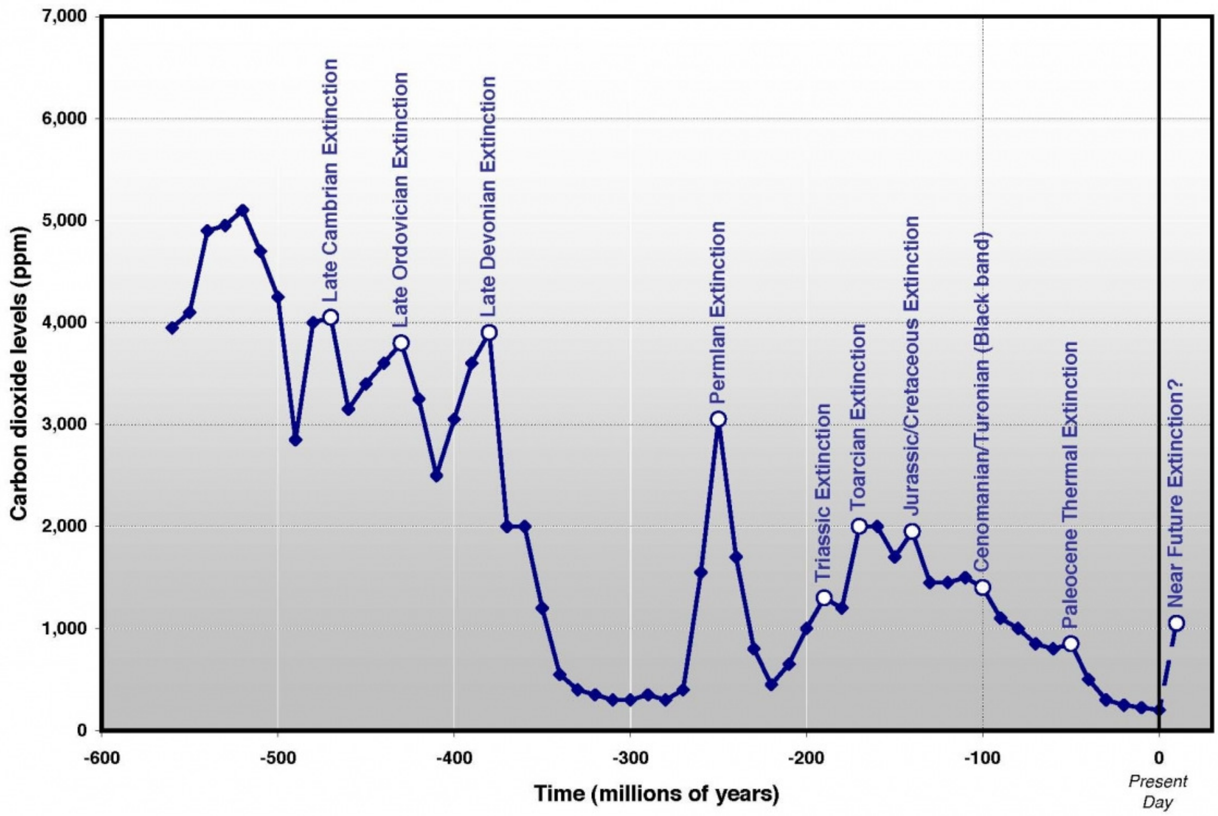
These small changes can affect Earth in a big way. For example, between 1645 and 1715 (a period astronomers call the "Maunder Minimum") the sunspot cycle stopped; the face of the Sun was nearly blank for 70 years. At the same time Europe was hit by an extraordinary cold spell: the Thames River in London froze, glaciers advanced in the Alps, and northern sea ice increased. An earlier centuries-long surge in solar activity (inferred from studies of tree rings) had the opposite effect: Vikings were able to settle the thawed-out coast of Greenland in the 980s, and even grow enough wheat there to export the surplus to Scandinavia.

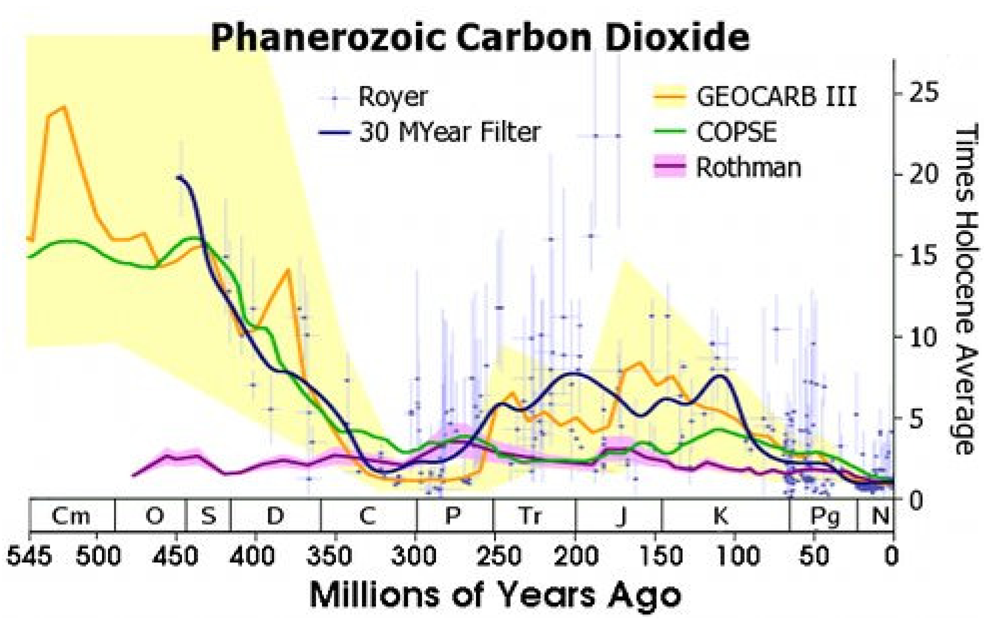


Above: Inferred variations in solar intensity (red and green lines) over the last 900 years appear to be related to the severity of winters in London and Paris. The red line is deduced from the abundance of a heavy form of carbon (carbon-14) in tree rings. This "isotope" of carbon is formed in the upper atmosphere when incoming cosmic rays smash into carbon dioxide molecules. When the Sun's activity is low, its weakened magnetic field lets more cosmic rays into the solar system, so carbon-14 abundances go up. (Notice on the graph that the scale for carbon-14 is upside down.) This image by scientist John Eddy is based on an earlier one that appeared in *Science*, 192, 1189 (1976)

Geological historical CO_2 levels

CO_2 levels from present to 550 million years ago as graphed by Dr. Peter Ward, correlated with mass extinctions that occur when CO_2 levels exceed 1000 ppm. The last "Snowball Earth" was 630 million years ago with the pre-Cambrian explosion of life occurring 100 million years later.





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